$$\therefore\ y_1=z+x\sin z+\frac{x^2}{1.2}\frac{d}{dz}(\sin^2 z)+\frac{x^3}{1.2.3}\left(\frac{d}{dz}\right)^2(\sin^3 z)+\text{ etc.}$$

$$\therefore\ y_1=z+x\sin z+\frac{x^2}{1.2}\frac{d}{dz}\left(\frac{1-\cos 2z}{2}\right)+\frac{x^3}{1.2.3}\left(\frac{d}{dz}\right)^2\left(\frac{3\sin z-\sin 3z}{4}\right)$$

$$+\frac{x^4}{1.2.3.4}\left(\frac{d}{dz}\right)^3\left(\frac{3-4\cos 2z+\cos 4z}{8}\right)+\text{ etc.}$$

$$=z+x\sin z+\tfrac{1}{2}x^2\sin 2z+\tfrac{1}{8}x^3(3\sin 3z-\sin z).$$

$$\therefore\ v=m+e\sin m+\tfrac{1}{2}e^2\sin 2m+\tfrac{1}{8}e^3(3\sin 3m-\sin m)$$

$$+\tfrac{1}{6}e^4(2\sin 4m-\sin 2m)+\text{ etc. }=m+e\sin v.$$

$$\therefore\ \sin v=\sin m+\tfrac{1}{2}e\sin 2m+\tfrac{1}{8}e^2(3\sin 3m-\sin m)+\tfrac{1}{6}e^3(2\sin 4m-\sin 2m)+\text{etc.}$$

To develop $(1-e\cos v)$ in terms of m: Let $f(y_1)=1-e\cos y_1$, $f'(y_1)=e\sin y_1$.

$$\therefore\ 1-e\cos y_1=(1-e\cos z)+x\sin z(e\sin z)+\frac{x^2}{1.2}\frac{d}{dz}(\sin^2 z.e\sin z)+\text{ etc.}$$

Performing the operations as before we get after substituting v for y_1, m for z, e for x,

$$1-e\cos v=1-e\cos m+\tfrac{1}{2}e^2(1-\cos 2m)+\tfrac{1}{8}e^3(3\cos m-3\cos 3m)$$

$$+\tfrac{1}{4}e^4(\cos 4m-\cos 2m)+\text{ etc.}$$

$$\therefore\ \cos v=\cos m+\tfrac{1}{2}e(\cos 2m-1)+(3e^2/8)(\cos 3m-\cos m)$$

$$+\tfrac{1}{4}e^3(\cos 2m-\cos 4m)+\text{ etc.}$$

$$\therefore\ \sin v=A\sin m+B\sin 2m+C\sin 3m+D\sin 4m+\ldots.$$
$$\cos v=-\tfrac{1}{2}e+A_1\cos m+B_1\cos 2m+C_1\cos 3m+D_1\cos 4m+\ldots.$$

where A, B, C, D, ...A_1, B_1, C_1, D_1, ... are each a series in powers of e.

$$\therefore\ x=a\cos v=-\tfrac{1}{2}ae+aA_1\cos m+aB_1\cos 2m+aC_1\cos 3m+aD_1\cos 4m+\ldots.$$
$$y=b\sin v=bA\sin m+bB\sin 2m+bC\sin 3m+bD\sin 4m+\ldots.$$
$$r=a(1-e\cos v)=a[1+\tfrac{1}{2}e^2-eA_1\cos m-eB_1\cos 2m-eC_1\cos 3m$$
$$-eD_1\cos 4m+\ldots.].$$

126. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

The declination of a certain fixed star if 12° 40′. Its altitude was observed one day to be 16° 40′. Three hours and twenty-four minutes later it was found to be 40° 20′. Find the latitude of the place of observation.

Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $\delta=12^\circ\ 40'$, φ=latitude of observation, $a=16^\circ\ 40'$, $a'=40^\circ\ 20'$, $\mu=51^\circ$ =3 hours, 24 minutes, h=hour angle.

$$\text{Then } \cos h=\frac{\sin a-\sin\varphi\sin\delta}{\cos\varphi\cos\delta}=x,\quad \cos(h-\mu)=\frac{\sin a'-\sin\varphi\sin\delta}{\cos\varphi\cos\delta}=y.$$

Eliminating h, $\sin^2\mu=x^2-2xy\cos\mu+y^2$. Substituting values of x and y,

$$\cos^2\varphi\cos^2\delta\sin^2\mu=2\sin^2\varphi\sin^2 d(1-\cos\mu)+\sin^2 a+\sin^2 a'-2\sin a\sin a'\cos\mu$$
$$-2\sin\varphi\sin\delta(1-\cos\mu)(\sin a+\sin a').$$

$$\therefore\ \sin^2\varphi[\sin^2\mu+\sin^2\delta(1-\cos\mu)^2]-2\sin\varphi\sin\delta(1-\cos\mu)(\sin a+\sin a')$$
$$=\cos^2\delta\sin^2\mu-\sin^2 a-\sin^2 a'+2\sin a\sin a'\cos\mu.$$

Let $\sin^2\mu+\sin^2\delta(1-\cos\mu)^2=A=.610569$.
$\sin\delta(1-\cos\mu)(\sin a+\sin a')=B=.075921$.
$\cos^2\delta\sin^2\mu-\sin^2 a-\sin^2 a'+2\sin a\sin a'\cos\mu=C=.307394$.

$\therefore\ A\sin^2\varphi-2B\sin\varphi=C$.

$\therefore\ \sin\varphi=\dfrac{B\pm\sqrt{(AC+B^2)}}{A}=.844702$, or $-.596013$. $\varphi=57^\circ\ 38'\ 37''$.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

166. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

If I sell one of my farms for $\$A$,=\$4500, and the other for $\$B$,=\$1800, I will gain $p\%$, =5%, on cost of both; but if I sell the dearer farm for $\$C$,=\$4000, and the other at cost, I will lose $p\%$,=5%. Find the cost of each farm.

AVERAGE AND PROBABILITY.

138. Proposed by G. B. M. ZERR, A. M., Ph. D, Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find the average area of (1) triangle, (2) quadrilateral, (3) pentagon, (4) hexagon, formed by taking (1) three, (2) four, (3) five, (4) six random points on the circumference of a given circle radius a.

139. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

Four points are taken at random on the surface of a given sphere; find the average volume of the tetrahedron formed by the planes passing through the points taken three and three.